Some estimates in a non self-adjoint context

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Holomorphic functions of one operator frequently occur in pure as applied mathematics. For instance, the exponential function is related to the semigroup, or the group theory, i.e. in particular to parabolic equations, linear Schroedinger, ... The cosine function plays a similar role for second order evolution equations. Time discretizations of these problems (for instance by Runge-Kutta methods) lead to use rational approximations of the exponential or of the cosine function. Explicit methods and Krylov type methods correspond with polynomial approximations. For the theory, as well as for the numerical analysis of these problems, estimates of these holomorphic functions are needed. In the case of a self-adjoint operator, spectral theory provides a very efficient tool, but the situation is much more complicated if we consider non normal operators. For them, spectral sets were introduced and studied by J. von Neumann in 1951, but this theory have not known a deep extension up to a work of B.&F. Delyon. In this talk I will review some results which have been inspired by their paper, and present some situations in which we have been able to improve the existing estimates.

References

[1] B.&F. Delyon, Generalization of Von Neumann's spectral sets and integral representation of operators, *Bull. Soc. Math. France*, **1**, (1999), 25–42.

[2] J. von Neumann, Eine Spektraltheorie für allgemeine Operatoren eines unitären Raumes, *Math. Nachr.* **4** (1951), 258–281.