

# Recent Applications of the Łojasiewicz gradient inequality

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The Łojasiewicz gradient inequality, initially devised to establish convergence to equilibrium of bounded solutions to the general gradient system

$$u' + \nabla F(u) = 0$$

when  $F$  is an analytic potential on  $\mathbb{R}^N$  with a continuum of equilibria led in the last decade to many new applications concerning both convergence and the rate of decay to equilibrium of various dynamical systems involving an analytic potential and having a certain type of Liapunov function. These systems include

i) Second order ODE of the form

$$u'' + g(u') + \nabla F(u) = 0$$

ii) Systems with rapidly decaying source terms

$$U' + \mathcal{M}U = H(t)$$

iii) Some discrete dynamical systems.

In this survey we shall try to depict the state of the art in both finite and infinite dimensional (PDE) frameworks and to outline the main open problems that emerged in this field of research.