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The stochastic Cahn-Hilliard equation with reflexions and singular terms.

Abstract: The aim of this talk is to present recent results obtained in collaboration with L. Zambotti and L. Goudenège on the Cahn-Hilliard equation :

$$\begin{cases} du = -\partial_{xx}\left(\partial_{xx}u + f(u) + \eta\right)dt + \partial_x dW, \\ u \text{ and } \partial_{xx}u \text{ satisfy Neumann boubary conditions,} \\ u(0) = u_0. \end{cases}$$

Here $u(t,x)$ is a random process defined for $(t,x) \in \mathbb{R}^+ \times [0,1]$, W is a cylindrical Wiener process - in other words $\frac{dW}{dt}$ is the space time white noise -, f is a nonlinear term and η is reflexion measure which enables to enforce the solution to live in a fixed set.

In a first work, we have studied the case $f = 0$ under the additional requirement that $u \geq 0$ and the contact condition $\int u d\eta = 0$. In the case of the heat equation, this problem is well understood. One of the main tool for its study is the comparison principle. This is no longer available for our fourth order equation. We have developped new methods to overcome this difficulty and obtained existence and uniqueness of solutions. This is the first step to justify the study of some interacting particle systems whose limiting behaviour is described by this equation.

Then, L. Goudenège, combining ideas from this article and previous works by L. Zambotti in the case of the heat equation, has studied the case $u \geq 0$ and $f(u) = -u^{-\alpha}$ or $f(u) = \ln u$. He has been able to prove existence and uniqueness. He has also shown that the reflexion measure vanishes if and only if $\alpha \geq 0$.

Finally, we have studied the case $f(u) = \ln\left(\frac{1-u}{1+u}\right) + \lambda u$ with the constraint $u \in [-1,1]$. In this case $\eta = \eta^+ - \eta^-$, where $\eta^\pm$ are two positive reflexion measures subjected to contact conditions at ± 1. This is the original model proposed by Cahn and Hilliard. The presence of two constraints introduces additional difficulties. We again prove existence and uniqueness. Furthermore, we prove ergodicity of the associated transition semigroup.